

# Chiral phase transition in an extended NJL model with higher-order multi-quark interactions

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## Abstract

The chiral phase transition is studied in an extended Nambu–Jona-Lasinio model with eight-quark interactions. Equations for scalar and vector quark densities, derived in the mean field approximation, are nonlinear and mutually coupled. The scalar-type nonlinear term hastens the restoration of chiral symmetry, while the scalar-vector mixing term makes the transition sharper. The scalar-type nonlinear term shifts the critical endpoint toward the values predicted by lattice QCD simulations and the QCD-like theory.

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Qualitative properties in quantum chromodynamics (QCD) at high temperatures and densities attract much attention. One of the most important recent findings is strong correlations in the quark gluon plasma just above the critical temperature; it is realized as the near perfect fluidity [1, 2, 3] and the mesonic correlations [4, 5].

With the aid of the progress in computer power, lattice QCD simulations have become feasible for thermal systems at zero or small density [6, 7]. At high density, however, lattice QCD is still not feasible due to the sign problem. As an approach complementary to the first-principle lattice simulations, we can consider several effective models. One of them is the Nambu–Jona-Lasinio (NJL) model. Since it was proposed [8], although this is a model of chiral symmetry that does not possess a confinement mechanism, this model has been widely used [9, 10] in the mean field approximation (MFA), for example, for analyses of the critical endpoint of chiral transition [11, 12, 13, 14, 15, 16].

Although the NJL model is recognized as an useful method for understanding the chiral symmetry breaking, only a few studies were done so far on roles of higher-order multi-quark interactions [17, 18], except for the case of the six-quark interaction coming from the 't Hooft determinant interaction [19]. The NJL model is an effective theory of QCD, so there is no reason, in principle, why higher-order multi-quark interactions are excluded. In the nonperturbative renormalization group method [20], such higher-order interactions are produced as a result of quantum effects in the high momentum region. Such effects should be included in the low-energy effective action from the beginning. Thus, it is quite meaningful to study effects of higher-order interactions on the chiral phase transition.

In this paper, we consider an extended NJL model that newly includes eight-quark interactions and analyze roles of such higher-order interactions on the chiral phase transition. It is well known that the original NJL model predicts a critical endpoint to appear at a lower temperature ( $T$ ) and a higher chemical potential ( $\mu$ ) than lattice QCD [7] and the QCD-like theory [21, 22] do. We will show in this paper that a scalar-type eight-quark interaction shifts the critical endpoint toward values predicted by the lattice simulations and the QCD-like theory.

As for the repulsive vector-type four-quark interaction  $(\bar{q}\gamma_\mu q)^2$ , it is well-known that it makes the chiral phase transition weaker in the low  $T$  and high  $\mu$  region, so there is a possibility that the transition becomes a crossover in the region when the interaction is strong enough [12, 16]. In this point of view, an *absence* of the vector-type four-quark

interaction may be preferable in the high density region; the absence is also supported by works of Refs. [23] and [24].

On the contrary, in the relativistic meson-nucleon theory [25], the repulsive force mediated by vector mesons is necessary to account for the saturation property of nuclear matter. Using the auxiliary field method, we can convert quark-quark interactions to meson-quark interactions; for example, see Refs. [26, 27, 28] and references therein. It is then natural to think that there exists a relation between the meson-nucleon interaction and the quark-quark interaction in the NJL model, since a nucleon is composed of three constituent quarks each of which has a large effective mass as a result of the spontaneous symmetry breaking (SSB) of chiral symmetry. In this point of view, the vector-type four-quark interaction  $(\bar{q}\gamma_\mu q)^2$  is *indispensable* around  $\mu \sim 308$  MeV corresponding to the normal nuclear density region. Thus, it is expected that the vector-type interaction is sizable in the normal density region but suppressed in the higher density region.

In the relativistic meson-nucleon theory, it is known that nonlinear meson-nucleon interactions suppress the effective coupling between mesons and nucleons in the higher density region [28, 29, 30, 31]. It is then strongly expected that a similar situation takes place in the NJL model as soon as higher-order multi-quark interactions are introduced. This is shown in this paper.

This is the first work to study roles of eight-quark interactions on phase transitions, so we concentrate our analysis on the chiral phase transition and use a simple model with two flavor quarks. Effects of the higher-order interactions on the color superconductivity will be discussed in a forthcoming paper.

We start with the following chiral-invariant Lagrangian density with two flavor quarks

$$\begin{aligned} \mathcal{L} = & \bar{q}(i \not{\partial} - m_0)q + \left[ g_{2,0} \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right) + g_{4,0} \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right)^2 \right. \\ & \left. - g_{0,2} (\bar{q}\gamma^\mu q)^2 - g_{2,2} \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right) (\bar{q}\gamma^\mu q)^2 + \cdots \right], \end{aligned} \quad (1)$$

where  $q$ ,  $m_0$  and  $g_{i,j}$  stand for a quark field, a bare quark mass and coupling constants. Here, we consider only four- and eight-quark interactions by ignoring the higher-order interactions denoted by dots in Eq. (1). Furthermore, we disregard interactions including isovector-vector current not important in symmetric quark matter, and also does the vector-type eight-quark interaction  $(\bar{q}\gamma_\mu q)^4$ , because the expectation value of the vector current  $\bar{q}\gamma_0 q$  is smaller than that of the scalar one  $\bar{q}q$ , unless the chemical potential is quite large. The mean

field approximation reduces the Lagrangian to

$$\begin{aligned}
\mathcal{L}_{\text{MFA}} = & \bar{q}(i\not{\partial} - m_0)q \\
& + \left[ 2g_{2,0} + 4g_{4,0} \left( \langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) - 2g_{2,2} \langle \bar{q}\gamma^\mu q \rangle^2 \right] \left( \langle \bar{q}q \rangle \bar{q}q + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle \bar{q}i\gamma_5 \vec{\tau}q \right) \\
& - \left[ 2g_{0,2} + 2g_{2,2} \left( \langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) \right] \langle \bar{q}\gamma^\mu q \rangle \bar{q}\gamma_\mu q \\
& - g_{2,0} \left( \langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) - 3g_{4,0} \left( \langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right)^2 \\
& + 3g_{2,2} \left( \langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) \langle \bar{q}\gamma^\mu q \rangle^2 + g_{0,2} \langle \bar{q}\gamma^\mu q \rangle^2.
\end{aligned} \tag{2}$$

Below, just for simplicity of the notation, we will omit terms including the pseudo-scalar current  $\bar{q}i\gamma_5 \vec{\tau}q$  and the spatial components  $\bar{q}\gamma_i q$  ( $i = 1, 2, 3$ ) of the vector current, since their expectation values vanish. It is convenient to introduce two auxiliary mean fields as

$$\sigma \equiv \langle \bar{q}q \rangle, \quad \omega \equiv \langle \bar{q}\gamma_0 q \rangle. \tag{3}$$

Using these auxiliary fields, one can rewrite  $\mathcal{L}_{\text{MFA}}$  as

$$\mathcal{L}_{\text{MFA}} = \bar{q}[i\not{\partial} - (m_0 + \Sigma_s) + \Sigma_v \gamma_0]q - U, \tag{4}$$

where

$$\begin{aligned}
\Sigma_s &= -\left( 2g_{2,0}\sigma + 4g_{4,0}\sigma^3 - 2g_{2,2}\sigma\omega^2 \right), \quad \Sigma_v = -\left( 2g_{0,2}\omega + 2g_{2,2}\sigma^2\omega \right), \\
U &= g_{2,0}\sigma^2 + 3g_{4,0}\sigma^4 - 3g_{2,2}\sigma^2\omega^2 - g_{0,2}\omega^2.
\end{aligned} \tag{5}$$

The thermodynamical potential  $\Omega$  of the system with finite  $T$  and  $\mu$  is then obtained by

$$\begin{aligned}
\Omega(T, \mu) = & -2N_f N_c V \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[ E_p + \frac{1}{\beta} \left( \log(1 + e^{-\beta(E_p - \mu^*)}) + \log(1 + e^{-\beta(E_p + \mu^*)}) \right) \right] \\
& + VU,
\end{aligned} \tag{6}$$

where  $\beta = 1/T$ ,  $\mu^* = \mu + \Sigma_v$ ,  $E_p = \sqrt{\mathbf{p}^2 + M^2}$  and  $M$  stands for the effective quark mass defined as  $M = m_0 + \Sigma_s$ . The corresponding scalar and vector quark densities,  $\rho_s$  and  $\rho_v$ , are also given by

$$\rho_s = 2N_f N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{M}{E_p} (n_q + n_{\bar{q}} - 1), \quad \rho_v = 2N_f N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} (n_q - n_{\bar{q}}), \tag{7}$$

where  $n_q = 1/[1 + \exp\{\beta(E_p - \mu^*)\}]$  and  $n_{\bar{q}} = [1 + \exp\{\beta(E_p + \mu^*)\}]$ . The physical solutions of  $\sigma$  and  $\omega$  satisfy the stationary condition

$$\begin{pmatrix} \frac{\partial}{\partial \sigma} \left( \frac{\Omega}{V} \right) \\ \frac{\partial}{\partial \omega} \left( \frac{\Omega}{V} \right) \end{pmatrix} = \mathcal{G}^* \begin{pmatrix} \sigma - \rho_s \\ \rho_v - \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathcal{G}^* \equiv \begin{pmatrix} G_{s\sigma}^* & G_{v\sigma}^* \\ G_{s\omega}^* & G_{v\omega}^* \end{pmatrix}. \tag{8}$$

Here we have defined four effective couplings,  $G_{s\sigma}^*$ ,  $G_{v\sigma}^*$ ,  $G_{s\omega}^*$  and  $G_{v\omega}^*$ , as [28, 29, 30, 31]

$$G_{s\sigma}^* \equiv -\frac{\partial \Sigma_s}{\partial \sigma} = 2g_{2,0} + 12g_{4,0}\sigma^2 - 2g_{2,2}\omega^2, \quad G_{v\sigma}^* = -\frac{\partial \Sigma_v}{\partial \sigma} = 4g_{2,2}\sigma\omega. \quad (9)$$

$$G_{s\omega}^* = -\frac{\partial \Sigma_s}{\partial \omega} = -4g_{2,2}\sigma\omega = -G_{v\sigma}^*, \quad G_{v\omega}^* = -\frac{\partial \Sigma_v}{\partial \omega} = 2g_{0,2} + 2g_{2,2}\sigma^2. \quad (10)$$

When  $\det(\mathcal{G}^*) = G_{s\sigma}^* G_{v\omega}^* + G_{v\sigma}^{*2} \neq 0$ , which is satisfied in our analyses below, the matrix  $\mathcal{G}^*$  has its inverse, and then the stationary condition (8) leads to  $\sigma = \rho_s$  and  $\omega = \rho_v$ , showing the consistency with Eq. (3) [30]. However, the solutions  $\sigma$  and  $\omega$  to the equations do not necessarily yield a minimum of  $\Omega$ . The solution  $\sigma$  is then determined by searching for minima of  $\Omega(\sigma, \omega(\sigma))$  in which  $\omega$  is eliminated with  $\omega = \rho_v$ .

Identifying  $\bar{q}i\gamma_5\vec{\tau}q$  with the pion field, we can define the effective coupling  $G_{s\pi}^*$  between pion and quark as

$$iG_{s\pi}^*\gamma_5\vec{\tau} \equiv \frac{\delta^3}{\delta q \delta \langle \bar{q}i\gamma_5\vec{\tau}q \rangle \delta \bar{q}} \mathcal{L}_{\text{MFA}} \Big|_{\langle \bar{q}i\gamma_5\vec{\tau}q \rangle=0} = i(2g_{2,0} + 4g_{4,0}\sigma^2 - 2g_{2,2}\omega^2) \gamma_5\vec{\tau}. \quad (11)$$

In the random phase approximation (RPA), the pion mass  $M_\pi$  at  $T = \mu = 0$  is determined with this effective coupling as

$$M_\pi^2 = \frac{4m_0}{G_{s\pi}^* M I(M, M_\pi)}, \quad I(x, y) = 8N_f N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{\mathbf{p}^2 + x^2} (4(\mathbf{p}^2 + x^2) - y^2)}. \quad (12)$$

Similarly, the  $\sigma$ -meson mass  $M_\sigma$  at  $T = \mu = 0$  is determined as

$$M_\sigma^2 = \frac{4m_0 + 32g_{4,0}\sigma^3}{G_{s\sigma}^* M I(M, M_\sigma)} + 4M^2. \quad (13)$$

This equation indicates that  $M_\sigma < 2M$  when  $\sigma < 0$  and  $g_{4,0} > 0$ .

Since the NJL model is nonrenormalizable, it is needed to introduce a cutoff in the momentum integration. In this paper, we use the three-dimensional momentum cutoff as

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \rightarrow \frac{1}{2\pi^2} \int_0^\Lambda p^2 dp. \quad (14)$$

Hence, the present model has six parameters,  $m_0$ ,  $\Lambda$ ,  $g_{2,0}$ ,  $g_{4,0}$ ,  $g_{0,2}$  and  $g_{2,2}$ . We simply assume  $m_0 = 5.5$  MeV. For the case of  $g_{4,0} = 0$ ,  $\Lambda$  and  $g_{2,0}$  are fixed to reproduce the empirical values of the pion decay constant  $f_\pi = 93.3$  MeV and the pion mass  $M_\pi = 138$  MeV. For the case of nonzero  $g_{4,0}$ ,  $\Lambda$ ,  $g_{2,0}$  and  $g_{4,0}$  are fixed to reproduce  $f_\pi$  and  $M_\pi$  above and  $M_\sigma = 650$  MeV. In the latter case, the contribution of the  $g_{4,0}$  term to  $G_{s\sigma}^*$  is about

11 percents at  $T = 0$  and  $\mu = 0$ . For the vector coupling constants  $g_{0,2}$  and  $g_{2,2}$ , we take two extreme cases,  $G_\omega = 0$  and  $G_\omega = G_\sigma/1.5$ , where  $G_\sigma \equiv G_{s\sigma}^*|_{T=\mu=0} = 2g_{2,0} + 12g_{4,0}\sigma_0^2$  and  $G_\omega \equiv G_{v\omega}^*|_{T=\mu=0} = 2g_{0,2} + 2g_{2,2}\sigma_0^2$  for  $\sigma_0$  the scalar density at  $T = 0$  and  $\mu = 0$ . Furthermore, in order to determine each value of  $g_{0,2}$  and  $g_{2,2}$ , we take two cases,  $(2g_{0,2}, 2g_{2,2}\sigma_0^2) = (G_\omega, 0)$  and  $(2g_{0,2}, 2g_{2,2}\sigma_0^2) = (0.8G_\omega, 0.2G_\omega)$ . For the second case,  $G_{v\omega}^*$  is suppressed in the high  $\mu$  region, as shown later. Table I summarizes the parameter sets we have taken,  $M_\sigma$  at  $T = 0$  and  $\mu = 0$ , and the type of transition at  $T = 0$ .

model	$g_{2,0}$	$g_{4,0}\sigma_0^2$	$2g_{0,2}$	$2g_{2,2}\sigma_0^2$	$M_\sigma$ [GeV]	type
NJL	5.498	0	0	0	0.681	first order
NJL + $\sigma^4$	5.276	0.1109	0	0	0.650	first order
NJL + $\omega^2$	5.498	0	$G_\omega$	0	0.681	crossover
NJL + $\omega^2 + \sigma^2\omega^2$	5.498	0	$0.8 G_\omega$	$0.2 G_\omega$	0.681	crossover
NJL + $\sigma^4 + \omega^2$	5.276	0.1109	$G_\omega$	0	0.650	crossover
NJL + $\sigma^4 + \omega^2 + \sigma^2\omega^2$	5.276	0.1109	$0.8 G_\omega$	$0.2 G_\omega$	0.650	first order

TABLE I: Summary of the parameter sets,  $M_\sigma$  at  $T = 0$  and  $\mu = 0$ , and the type of the transition at  $T = 0$ . The coupling constants are shown in  $\text{GeV}^{-2}$ . The effective coupling  $G_\omega$  is fixed to  $7.331 \text{ GeV}^{-2}$  in the NJL+ $\omega^2$  and the NJL+ $\omega^2+\sigma^2\omega^2$  model and to  $7.922 \text{ GeV}^{-2}$  in the NJL+ $\sigma^4+\omega^2$  and the NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$  model. For all cases, we take  $\Lambda = 0.6315 \text{ GeV}$  and  $M|_{T=\mu=0} = 0.3379 \text{ GeV}$  and  $\sigma_0 = -0.03023 \text{ GeV}^3$ .

Figure 1(a) shows the  $T$  dependence of the effective quark mass for the case of  $\mu = 0$ . Since the  $\omega$  field has no contribution to  $M$  when  $\mu = 0$ , results are shown only for the original NJL and the NJL+ $\sigma^4$  model. The nonlinear  $\sigma^4$  interaction makes the restoration of chiral symmetry faster, since the effective coupling, responsible for the SSB of chiral symmetry, becomes smaller as  $T$  increases, as shown in Fig.1(b). However, the interaction keeps the transition a crossover as in the case of the original NJL. The NJL+ $\sigma^4$  model yields a smaller transition temperature ( $T_c \sim 180 \text{ MeV}$ ) than the original NJL model does ( $T_c \sim 190 \text{ MeV}$ ). The value,  $T_c \sim 180 \text{ MeV}$ , is close to the one predicted by the lattice simulation ( $T_c \sim 170 \text{ MeV}$ ) [7].

Figure 2 shows the effective quark mass as a function of  $\mu$  with  $T = 0$ . In the original NJL model, the chiral transition is a first order. The nonlinear  $\sigma^4$  interaction makes the transition faster, as shown in the result of the NJL+ $\sigma^4$  model.

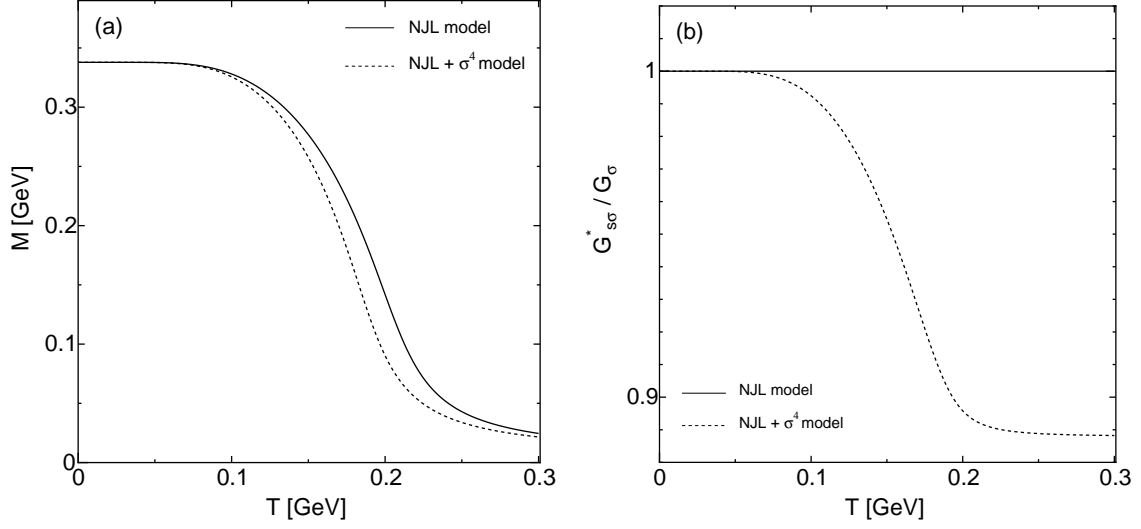


Fig. 1: The  $T$  dependence of (a) the effective quark mass and (b) the effective coupling in the case of  $\mu = 0$ .

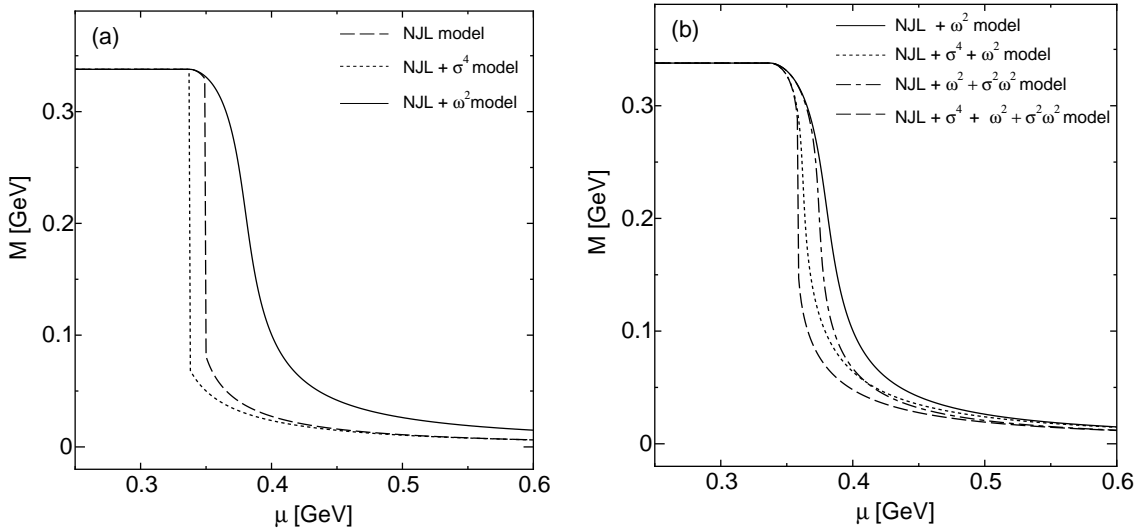


Fig. 2: The  $\mu$  dependence of the effective quark mass in the case of  $T = 0$ .

As already shown in Ref. [12, 16], the  $\omega^2$  interaction tends to change the transition from a first order to a crossover; actually this is seen in the result of the NJL +  $\omega^2$  model of Fig. 2(a). As an interesting result, however, the nonlinear  $\sigma^4$  and  $\sigma^2\omega^2$  interactions make the transition sharper again, so that the transition returns to a first order in the full-fledged

NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$  model. Thus, the nonlinear  $\sigma^4$  and  $\sigma^2\omega^2$  interactions work so as to cancel out the well-known effect of the  $\omega^2$  interaction.

Figure 3 shows the effective couplings as functions of  $\mu$  for the case of  $T = 0$ . For all models, both  $G_{s\sigma}^*$  and  $G_{v\omega}^*$  are suppressed in the high  $\mu$  region, but each model has its own  $\mu$  dependence. In the NJL+ $\sigma^4$  model,  $G_{s\sigma}^*$  decreases suddenly in the high  $\mu$  region and then approaches the value of  $2g_{2,0}$ , because the  $\sigma$ -dependent part of  $G_{s\sigma}^*$  almost vanishes there. Similarly, in the NJL+ $\sigma^4+\omega^2$  model,  $G_{s\sigma}^*$  decreases rather rapidly but not suddenly as  $\mu$  increases and approaches the value of  $2g_{2,0}$ . The change in the  $\mu$  dependence of  $G_{s\sigma}^*$  from the the NJL+ $\sigma^4$  model to the NJL+ $\sigma^4+\omega^2$  model comes from the fact that the  $\omega^2$  interaction makes the phase transition weak. In the NJL+ $\omega^2+\sigma^2\omega^2$  model,  $G_{s\sigma}^*$  decreases gradually as  $\mu$  increases, because the  $\omega$ -dependent part of  $G_{s\sigma}^*$  gives a negative contribution to the effective coupling. In the full-fledged NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$  model,  $G_{s\sigma}^*$  is suppressed suddenly by the  $\sigma^4$  interaction and then suppressed gradually by the  $\sigma^2\omega^2$  mixing interaction.

As for  $G_{v\omega}^*$  shown in Fig. 3(b), one can see a similar sharp suppression but not find any gradual decrease. This is understandable from the fact that in Eq. (10) the coupling has a  $\sigma$ -dependent term but no  $\omega$ -dependent one. As a point to be noted, the sharp suppression comes from the  $\sigma^2\omega^2$  interaction for the case of  $G_{v\omega}^*$  but the  $\sigma^4$  interaction for the case of  $G_{s\sigma}^*$ . Furthermore, note that the  $\sigma^2\omega^2$  interaction yields both a gradual decrease of  $G_{s\sigma}^*$  and a sharp suppression of  $G_{v\omega}^*$ .

The  $\mu$  dependences of the effective couplings mentioned above are similar to that of the chiral symmetry restoration shown in Fig. 2. We can then consider that effects of higher-order interactions on the chiral symmetry restoration are described mainly through the effective couplings in their  $\mu$  dependences, although  $U$  is also changed by the higher-order interactions.

Figure 4 shows the phase diagram in the  $\mu$ - $T$  plane. Results are shown for the three cases of the original NJL model, the NJL+ $\sigma^4$  model and the NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$  model in which the transitions are first order in the high  $\mu$  and low  $T$  region. First-order transitions take place on the curves in Fig. 4, and at the endpoint  $(\mu_e, T_e)$  of each curve the transition is changed into a crossover. The NJL+ $\sigma^4$  model yields smaller  $\mu_e$  and larger  $T_e$  than the original NJL model does;  $(\mu_e, T_e) = (308, 54)$  MeV for the former and  $(330, 47)$  MeV for the latter. When we take another parameter set,  $\Lambda = 0.6315$  GeV,  $g_{2,0} = 5.00$  GeV $^{-2}$  and  $g_{4,0} = 271$  GeV $^{-8}$ , that reproduces  $M_\sigma = 600$  MeV, the endpoint of the NJL+ $\sigma^4$  model



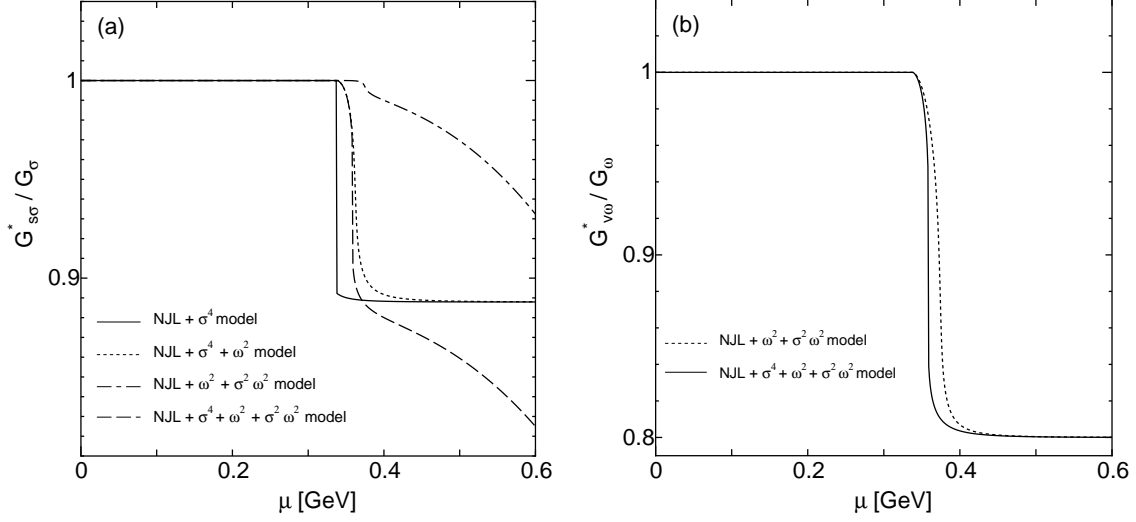


Fig. 3: Effective couplings as functions of  $\mu$  in the case of  $T = 0$ .

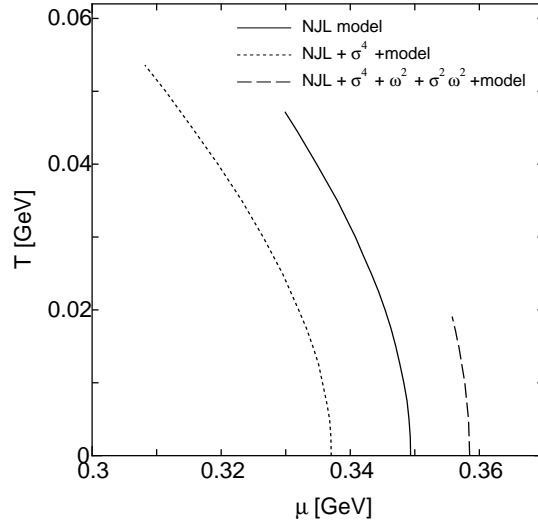


Fig. 4: Phase diagram in the  $\mu$ - $T$  plane. Each curve denotes the location of the first-order phase transition in each model.

is at (276, 62) MeV. Thus, the nonlinear  $\sigma$  interaction shifts the critical endpoint toward values predicted by the lattice QCD calculations,  $(\mu_e, T_e) = (242, 160)$  MeV [7], and by the QCD-like theory,  $(\mu_e, T_e) \sim (200, 100)$  MeV [21, 22].

As mentioned above, the  $\omega^2$  interaction tends to change the transition from a first order to a crossover, but this effect is partially canceled out by the scalar-vector mixing interac-

tion  $\sigma^2\omega^2$ . Consequently, as shown in Fig. 4, there exists a critical endpoint also in the NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$  model. The comparison of our models with the results of the lattice simulations and the QCD-like theory indicates that the NJL+ $\sigma^4$  model is most preferable among our models. This may imply that the cancellation is essential.

In summary, we have studied effects of eight-quark interactions on the chiral phase transition. The scalar-type nonlinear interaction  $\sigma^4$  hastens the restoration of chiral symmetry and shifts the critical endpoint toward the values predicted by the lattice simulations and the QCD-like theory. The vector-scalar mixing interaction  $\sigma^2\omega^2$  can make the transition sharper in the high  $\mu$  and low  $T$  region, while the  $\omega^2$  interaction works in the opposite direction. Thus, the effect of the mixing interaction tends to cancel out that of the  $\omega^2$  interaction in the high  $\mu$  region, while the  $\omega^2$  interaction is still strong in the normal nuclear density region. The roles of the eight-quark interactions are well understood through the effective couplings,  $G_{s\sigma}^*$ ,  $G_{v\sigma}^*$ ,  $G_{s\omega}^*$  and  $G_{v\omega}^*$ , in their  $\mu$  dependences. Our results indicate that eight-quark interactions are very important for phase transitions and must be studied in detail. It is very interesting to study effects of the interactions on a color superconductivity itself and its correlation with the chiral phase transition. The study is now in progress.

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